Hierarchical Bayesian Estimation of Mixed Hazard Models

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Summary

A variety of uncertainties affect the deterioration process of infrastructure. The heterogeneity of structures causes the problem of overdispersion of the deterioration rate. In order to overcome this problem, the mixed Markov deterioration hazard model has been proposed considering the heterogeneity of deterioration rate among groups of infrastructures. In this study, it is assumed that the overdispersion depends on the heterogeneity of the deterioration rate among groups of infrastructures. Then, the mixed Markov deterioration hazard model that takes into account hierarchical heterogeneity is formulated, and a hierarchical Bayesian estimation method is proposed.

Keywords: hierarchical Bayesian estimation, mixed Markov hazard model, heterogeneity

1. Introduction

In recent years, in the field of management of civil infrastructure, development of statistical deterioration prediction technology is remarkable. In particular, not only prediction of average deterioration of civil infrastructure but deterioration prediction of the institution group was attained by development of the mixed Markov deterioration hazard model by Obama *et al*. These have proposed the technique of estimating the mixed Markov deterioration hazard model with a stepwise maximum likelihood method. However this method has the problem of overdispersion. To overcome this overdispersion problem, in this study, when the mixed Markov deterioration hazard model is estimated using hierarchical Bayesian estimation, it is shown that the deterioration prediction which expressed the actual phenomenon correctly is possible.

2. Methodology

In the mixed Markov hazard model, an exponential hazard model is assumed to discrete the deterioration process of each state. The infrastructure to be analysed is divided into H institution groups. The institution group $h\ (h=1,\dots,H)$ consists of L_h institutions. The heterogeneity parameter ε^h denotes the heterogeneity of the hazard rate of group h. The hazard rate $\lambda_i^{l_h}$ in state $i\ (1=1,\dots,I-1)$ of institution $l_h\ (l_h=1,\dots,L_h)$ is,

$$\lambda_i^{l_h}=\tilde{\lambda}_i^{l_h}\varepsilon^h \quad (i=1,\dots,I-1;h=1,\dots,H;l_h=1,\dots,L_h) \quad (1)$$

where the $\tilde{\lambda}_i^{l_h}$ is the average hazard rate of institution l_h of group h in state i, and the heterogeneity parameter ε^h is always non-negative $\left(0<\varepsilon^h<\infty\right)$ parameter. A Gamma distribution is assumed as the prior distribution of $\boldsymbol{\varepsilon}$, and ϕ is configured as hyper parameter. Hence, probability that the state of the institution l_h is i in both times $\tau_A^{l_h}$ and $\tau_B^{l_h}=\tau_A^{l_h}+z^{l_h}$ is,

$$\pi_{ii}(z^{l_h})=\exp(-\tilde{\lambda}_i^{l_h}\bar{\varepsilon}^h z^{l_h}) \quad (2)$$

Markov transition probability of passing from state i to state j in z^{l_h} is given by,

$$\pi_{ij}(z^{l_h})=\sum_{s=i}^{j}\prod_{m=i,\neq s}^{j-1}\frac{\tilde{\lambda}_m^{l_h}}{\tilde{\lambda}_m^{l_h}-\tilde{\lambda}_s^{l_h}}\exp(-\tilde{\lambda}_s^{l_h}\bar{\varepsilon}^h z^{l_h}) \quad (i=1,\dots,I-1;j=i,\dots,I-1;h=1,\dots H) \quad (3)$$

Then, MCMC method which is a kind of hierarchical Bayesian estimation is used to estimate the parameters of the mixed Markov hazard model. It is the feature of the Bayesian estimation for an estimation result to be able to be found not as a value but as distribution.

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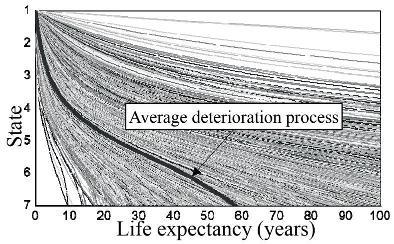

Fig. 1: Deterioration process of all slabs

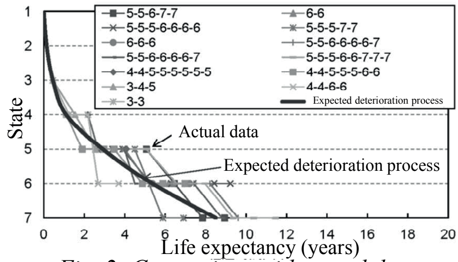

Fig. 2: Comparison with actual data

3. Empirical study

3.1 Application data

In order to examine the validity of the methodology proposed by this research, a hierarchical Bayesian estimation of the mixed Markov deterioration hazard model is tried using the visual check data of the bridge manages by the N City. A reinforced concrete flat slab is chosen as an object of estimation. The group of the reinforced concrete flat slab which sets a heterogeneity parameter is set to the bridge. The sum of the number of groups is 1,481.

Table 1: Estimated model parameters

	state	Absolute term $\beta_{i,1}$	Average traffic $\beta_{i,2}$	Size of slab $\beta_{i,3}$
Expected value Geweke's Z-score	1	-0.241 (-0.065)	-	1.567 (-0.342)
Expected value Geweke's Z-score	2	-1.027 (-0.103)	-	3.121 (0.193)
Expected value Geweke's Z-score	3	-1.794 (0.107)	0.677 (-0.150)	-
Expected value Geweke's Z-score	4	-2.827 (0.177)	1.122 (-0.202)	-
Expected value Geweke's Z-score	5	-3.087 (0.242)	-	-
Expected value Geweke's Z-score	6	-3.464 (0.312)	3.568 (-0.202)	-
Expected value Geweke's Z-score	Hyper parameter φ	1.096 (2.460)		

By using the mixed Markov deterioration hazard model, the deterioration prediction in consideration of structural or environmental conditions is attained. The hazard rate of a reinforced concrete flat slab of the group *k* (*k*=1,…,1,481) can be written as follows.

$$\lambda_i^h = \exp\left(\beta_{i,1} + \beta_{i,2}x_2^h + \beta_{i,3}x_3^h\right)\varepsilon^h \quad (5)$$

where, x_2^h = average traffic, x_3^h = size of slab, ε^h = heterogeneity parameter of the group h

3.2 Estimation result

In Table 1, the estimation result of the parameters and Geweke's Z-scores are indicated. In order to clarify the deterioration process of reinforced concrete flat slab, it is necessary to compute the expected life between each rating. The expected life of the reinforced concrete flat slab is computable using the estimated parameter. The hazard rate of group *h* is indicated by Eq.(1). By using hazard rate of group *h*, the life expectancy of the reinforced concrete flat slab of the group *h* is computable, respectively. The deterioration process of all the groups is shown in Fig.1. The minimum life is about ten years and the largest life is 100 years or more. In Fig.2, actual data and the data computed by estimation are plotted. We can compare the actual data and the expected deterioration process by estimating when we know the year which started the actual use of the bridge turns out that the estimation result is expressing the actual phenomenon with sufficient accuracy.

4. Conclusion

We propose the mixed Markov deterioration hazard model using hierarchical Bayesian estimation. If the method proposed by this study is used, high-precision deterioration prediction of infrastructure is possible. The method proposed by this study is applicable to deterioration prediction of various types of infrastructure. Furthermore, evaluation of the repair effect is possible by comparison of a heterogeneity parameter. Subdivision of a heterogeneity parameter can be raised as a future issue.