## Three-dimensional Parabolic Cable Element for Cable-Supported Structures

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## Summary

This paper presents a three-dimensional parabolic cable element for cable structures. The formulas of the diagonal and off-diagonal terms in the stiffness matrix are derived by using the assumption that the derivative of the total length of the cable is equal to that of the elastic elongation and the relationship between the nodal forces. Then, the proposed element is verified by using finite element analysis for several numerical examples. Since the verification results show that the proposed element offers a high degree of accuracy and computational efficiency relative to the catenary element, the proposed element is expected to be used not only in research but also for the practical design of cable-supported structures such as suspension bridges and cable-stayed bridges.
Keywords: Cable element; Parabola; Catenary; Stiffness matrix.

## 1. Introduction

Cables in cable-supported long-span bridges and cable roof structures have highly nonlinear behavior due to their low weight and stiffness. Therefore, their behaviors have been a topic of many studies. In general, cables are analyzed using two different approaches that regard a cable as a catenary or a parabola. The catenary as an exact approach for a cable requires a complicated iterative method to determine the nodal forces in order to obtain the stiffness matrix. Moreover, the existing parabola approach, Ernst's formula as a very simple and convenient method, cannot present accurate behavior for a slack cable. Therefore, a simple and accurate method to represent a cable is needed. In this paper, a three-dimensional parabolic cable element for cable structures is presented. The formulas of the stiffness matrix are derived by using the assumption that the derivative of the total length of the cable is equal to that of the elastic elongation and the relationship between the nodal forces. Then, the proposed element is verified by using finite element analysis for several numerical examples of cable structures subjected to static loadings.

## 2. Three-dimensional parabolic cable element



Fig. 1: Three-dimensional parabolic cable

Figure 1 shows the cable configuration considered in this study. Assuming a parabolic cable, the weight is distributed uniformly along the horizontal direction of the cable and the ratio of the sag $f$ at the midpoint to horizontal length $L_{12}$ is kept relatively small, that is, $1 / 8$ or less. Furthermore, it is assumed that the cross-sectional area, elastic modulus, and weight density are constant along its length, and the cable is under a small strain. In this figure, $q$ is the horizontally distributed load; $w$ the weight per unit length; $S_{0}$ the unstressed length; and $H$ the horizontal force at node $j$. Moreover, $F$ and $u$ indicate the nodal force and displacement at node $j$, respectively, and $F^{\prime}$

