Size Effect: What Is Its Rationale and Penalty for Neglect

Zdeněk P., Bažant
Professor
Northwestern University
Evanston, U.S
z-bazant@northwestern.edu

Qiang Yu
Postdoctoral Fellow
Northwestern University
Evanston, U.S
qiangyu@northwestern.edu

Summary

Concrete is a typical quasi-brittle material, which inevitably exhibits size effect on the nominal structural strength. This study is focussed on the size effect in shear failure of reinforce concrete beams. By statistical analysis it is shown that if the size effect is ignored, the failure frequency (or probability) of large reinforced beams under shear may typically increase by three orders of magnitude. By computer simulations based on fracture mechanics it is further shown that shear reinforcement cannot eliminate the size effect in large reinforced concrete beams.

Keywords: size effect, structural safety, reinforced concrete, shear failure, fracture mechanics, statistical analysis, design codes, legal risk.

1. Introduction

Concrete is an archetypical quasi-brittle material whose fracture propagation is characterized by a rather large fracture process zone. This causes that small structures fail in a quasi-ductile manner and exhibit almost no size effect, while very large structures failing in concrete rather than steel behave in an almost perfectly brittle manner and exhibit the strongest possible size effect. Although strong size effects occur in many types of failure in reinforced concrete, this study is limited in scope to the shear of slender concrete beams.

2. Risk of Failure in Shear Design

The ACI Building Code currently specifies the contribution of concrete to the cross-section shear strength of reinforced concrete members by the formula $V_c = 2\sqrt{f_c^{'}}b_w d$, which gives a size-independent concrete shear strength $v_c = V_c / b_w d = 2\sqrt{f_c^{'}}$. However, ignoring the size effect in this formula would lead to statistically dangerous designs with insufficient margins for large shear-critical beams.

To decide which data to use as an empirical basis for choosing the probability density function (pdf) of the beam shear strength, one should note that the ACI-445F database of 398 points has a downward trend with respect to beam depth *d*. Therefore, the entire database cannot be treated as a statistical population from which the pdf of shear strength could be identified. However, if one isolates the data in the small size range of depths *d* ranging from 10 to 30 cm, the size effect trend is weak enough for treating the data as a population with no statistical trend. By plotting the points from this small size range in cumulative histograms on various types of probability paper, or alternatively examining the type of pdf by goodness-of-fit tests, one finds that, among simple distributions, the log-normal pdf is the best for the small beam data in the ACI-445F database.

When the same log-normal pdf is superposed on the series of individual tests of beams of various sizes made at the University of Toronto, it should be noted that, for the type of concrete, steel ratio, shear span ratio, etc., used in the Toronto tests, the shear strength value in these tests lies (in the logarithmic scale) at certain distance *a* below the mean of the pdf. Since the width of the scatter band in the logarithmic scale does not vary appreciably with the beam size, the same pdf and the same distance *a* between the pdf mean and the Toronto data may be expected for every beam size *d*, including the size of *d* = 925 mm, for which there is only one data point. In other words, if the Toronto test for *d* = 925 mm were repeated for many different types of concrete, steel ratios, shear span ratios, humidity and temperature conditions, etc., one would have to expect a lognormal pdf with the same coefficient of variation, but with downward shift *a* in the logarithmic scale.

After determining the pdf of shear strength, and using a typical pdf of load, one can calculate the failure probability P_f of the beam. When P_f is calculated for the small beams within the range of depths *d* from 10 cm to 30 cm, and also for the large beams of 1 m depth, one obtains the following

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failure probabilities: $P_f \approx 10^{-6}$ for beams 0.2 m deep, and $P_f \approx 10^{-3}$ for beams 1 m deep. The failure probability of 10^{-6}, i.e., one in a million, obtained for small beams, corresponds to what the risk analysis experts generally consider as the maximum acceptable for engineering structures in general, because it does not appreciably add to the inevitable risks that people face anyway.

So, if the size effect in beam shear were ignored for beams without stirrups up to 1 m deep, the probability of failure for the 1 m depth would be about 1000-times greater than for the 20 cm depth. This would be unacceptable. If there should be any difference, it should be in the opposite sense because, for large beams, the failure consequences are usually more serious than for small ones.

3. Size Effect for Concrete Beams with Stirrups

Although there is little information on the size effect in shear failure of beams with minimum or heavier shear reinforcement, computational simulations, and even the limited experimental evidence that exists, reveal that stirrups do not eliminate the size effect but only mitigate it. Bažant’s energetic size effect law remains valid and the effect of stirrups is to increase the transitional size d_0 (intersection of size effect asymptotes). Avoidance of size effect would require eliminating the post-peak softening on the load-deflection diagram, and this could be achieved only if the concrete were subjected to triaxial confinement with all negative principal stresses exceeding in magnitude several times the uniaxial compression strength.

For slender beams with shear-span ratio $a/d > 2$, two test series are found in the literature: 1) the tests conducted by Bhal in 1968 in Stuttgart; and 2) the tests conducted by Kong and Rangan in 1998 in Perth. All the tests were made on slender beams with stirrups heavier than minimum requirement. In the logarithmic size effect plot, it can be seen clearly that the shear strength markedly decreases with increasing beam depth. The asymptotic size effect trend of slope -1/2 does not contradict these test results.

Finite element simulations based on the crack band model and the microplane model were also carried out to investigate geometrically similar beams of depths 0.47 m, 1.89 m and 7.56 m. Compared with the concrete without stirrups, the transitional size d_0 obtained is significantly increased. The simulations document that a strong size effect exists also in the beams with stirrups, although it is pushed into larger sizes.

4. Catastrophic Collapses in which Size Effect Played a Role

In the case of catastrophic sinking of Sleipner oil platform in a Norwegian fjord in 1991, which was due to shear failure of a thick tricell wall, there were three simultaneous mistakes. Beside two mistakes recognized by government forensic committee, Bažant pointed out that the size effect must have reduced the strength by ~34%, but this was omitted from the committee conclusions.

Of major interest for the size effect theory is the 1996 collapse of the Koror-Babeldaob Bridge in the Republic of Palau. In addition to the erroneous initial prediction of creep and shrinkage deflections and apparently inappropriate remedial prestressing, one would have to expect a major strength reduction due to size effect on the compression-shear failure seen in the photographs.

5. Concern to Concrete Societies: Legal Exposure

A quarter century ago, when the experimental data were scant and scattered, and only a handful of scientists espoused a coherent scientific theory, it was entirely plausible and defensible for the concrete societies to ignore the size effect. But by now the experimental evidence has become undeniable and the theoretical basis solid. Virtually all the researchers in fracture mechanics of concrete and entire research-oriented societies and committees in this field have no doubt that a significant non-statistical size effect exists in all the brittle failures of concrete structures. Consequently, ignoring size effect is no longer acceptable. It might expose concrete engineering societies to legal liability when another catastrophe occurs.

6. Conclusion

At the dawn of this century, the size effect in brittle failures of concrete structure has become an established fact. It is time to introduce it into the design codes and practice. Ignoring it will cause large structures to be failing with the frequency of about one per thousand or more, instead of less than one per million as generally considered tolerable for engineering structures. The human society must not be knowingly exposed to such a risk.