A Markov Model to Determine Optimal Intervention Strategies for Multiple Objects Affected by Uncorrelated Manifest and Latent Processes

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Summary

In many cases under regular inspection strategies, a sudden failure of the ability of infrastructure to provide an adequate level of service may occur due to processes, such as those associated with natural hazards, e.g. scouring during a flood, earth movements during an earthquake. In order to evaluate intervention strategies for infrastructure that is affected by such latent processes it is necessary to include the probability of having this inadequate level of service. This paper presents a Markov model that can be used to determine optimal intervention strategies for multiple objects affected by uncorrelated manifest and latent processes. The use of the model is demonstrated using a road link comprised of a road and a bridge.

Keywords: Markov model, infrastructure management, latent deterioration process, optimization.

1. Introduction

The subject of risk consideration, i.e. the deterioration of infrastructure that happens quickly due to such things as earthquakes and floods, has recently attracted a great attention in the field of infrastructure management [1]. This has been principally done by concentrating on the best ways to take into consideration future uncertainties, in particular with respect to the deterioration of the infrastructure, in the decision making process. Most of this work has, however, been concentrated on manifest deterioration processes, i.e. latent processes are not taken into consideration.

Latent processes are processes that are unobservable with under the inspection strategies being followed, so that there is enough time for a manager to execute an intervention so ensure that the infrastructure provides an adequate level of service. In this paper, a Markov model is proposed to determine optimal intervention strategies (OISs) for multiple objects affected by uncorrelated manifest and latent processes. One difference between this model and those currently used are the use of an extended set of condition states (CSs) to encompass those in which an adequate service level is provided and those in which an adequate service level is not provided, the latter for which the transition probabilities are estimated using normalized fragility curves. Another difference is the linear optimization program, which has been developed to take into consideration multiple objects simultaneously.

2. The models

When an object is only affected by manifest processes a multi-stage exponential Markov model is often used to model deterioration [2]. The following equation describes the explicit mathematical formula for estimating Markov transition probability (m.t.p) p_{ij} (details of formulation is referred to Tsuda et al, 2005 [2])

$$p_{ij}(z) = \sum_{k=i}^{j} \prod_{m=i}^{k-1} \frac{\theta_m}{\theta_m - \theta_k} \prod_{m=k}^{j-1} \frac{\theta_m}{\theta_{m+1} - \theta_k} \exp(-\theta_k z) \qquad (1)$$

Where, θ_i is hazard rate of each CS; i, j, k, m are running indexes of CS.

In order to integrate latent processes into the Markov model, it is therefore necessary to define additional CSs, i.e. ones that account for the states of objects when adequate service levels are not provided. Eq (2) shows the extensions of CSs to allow the consideration of latent process in our model.

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$$Q = \left[\begin{array}{cccc|cccc} p_{11} & p_{12} & \cdots & p_{1I} & e^p_{11} & e^p_{12} & \cdots & e^p_{1L} \\ 0 & p_{22} & \cdots & p_{2I} & e^p_{21} & e^p_{22} & \cdots & e^p_{2L} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & p_{II} & e^p_{I1} & e^p_{I2} & \cdots & e^p_{IL} \\ \hline 0 & 0 & \cdots & 0 & e_{11} & e_{12} & \cdots & e_{1L} \\ 0 & 0 & \cdots & 0 & 0 & e_{22} & \cdots & e_{2L} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 1 \end{array}\right] \quad (2)$$

In Eq. (2), p represents the m.t.p between the CSs where the object provides an adequate level of service, e^p represents the transition probabilities from the CSs where the object provides an adequate level of service to the CSs where the object does not provide an adequate level of service, and e represents between the CSs where the object does not provide an adequate level of service. The summation of the m.t.p between all CSs in each row equals 1.

The optimization model used in this work is a linear optimization model and is based on that proposed by Mayet and Madanat (2002) [1] for determining the OIS for single bridge affected by earthquakes. Some adaptions have been made to the model so that it can be used to take into consideration objects of multiple types. The objective of the model is to define OISs (represented by obtained steady state probability and corresponding intervention actions π_{n,a_n,i_n}) by minimizing total impacts (Eq. (3)) possibly incurred to stakeholders (e.g. owner, users, neighbours) subjected to a set of constraints (e.g. budgets, non-negative, balance) shown in Eqs. (3)-(9).

Objective function

$$MIN\sum_{n=1}^{N}\sum_{a_n=1}^{A_n}\sum_{i_n}^{I_n+L_n}\pi_{n,a_n,i_n}\cdot S_{n,a_n,i_n} \quad (3)$$

Constraints

$$S_{n,a_n,i_n}=\sum_{s=1}^{S}c^s_{n,a_n,i_n} \quad (4)$$

$$\sum_{n=1}^{N}\frac{1}{1-E_n}\sum_{a_n=1}^{A_n}\sum_{n=1}^{I_n}\pi_{n,a_n,i_n}\cdot c^s_{n,a_n,i_n}\le B^s \quad (5)$$

$$\pi_{n,a_n,i_n}\ge 0,\ \forall \mathrm{n}, \forall a_n, \forall i_n \quad (6)$$

$$\sum_{a_n=1}^{A_n}\sum_{i_n=1}^{I_n+L_n}\pi_{n,a_n,i_n}\cdot Q_{n,a_n,i_n,j_n}=\sum_{a_n=1}^{A_n}\pi_{n,a_n,j_n} \quad \forall j_n \quad (7)$$

$$\sum_{a_n=1}^{A_n}\sum_{i_n=1}^{I_n+L_n}\pi_{n,a_n,i_n}=1 \quad (8)$$

$$\pi_{n,a_n,i_n}=0 \quad \text{at} \quad a_n=1...(A_n\text{-}1) \text{ and } i_n=l_n \quad (9)$$

In the model, n, a_n, c, E_n, π_n, B, represents objects, intervention action, impacts, failure probability due to latent process, and budget, respectively.

3. Examples

The OIS is determined for a road link comprised of two objects, a road section and a bridge. Two failure states are considered for both road sections and bridges. Also, two budget scenarios are investigated: one is sufficient budget and the other is limited budget.

Given the m.t.p, intervention effectiveness, impacts incurred by performing interventions, and failure probability for each object, we use the model to obtain OISs for the two scenarios. It concludes from the estimation results that incorporating latent process in Markov models significantly affects the outcome OISs.

4. Conclusion

In this paper a methodology to determine OIS for objects affected by uncorrelated manifest and latent processes was presented. One difference between this model and those currently used are the use of an extended set of CSs to encompass those in which an adequate service level is provided and those in which an adequate service level is not provided the latter for which the transition probabilities are estimated using fragility curves. Another difference is the linear optimization program, which has been developed to take into consideration multiple objects. The model is demonstrated by determining the OIS for a road link comprised of road sections and bridges.

5. References

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