



## An extended sandwich theory for prestressed concrete bridges with corrugated steel web(s)

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### Summary

Prestressed concrete bridges with corrugated steel web(s) have emerged as a promising bridge form. In view of the negligible axial stiffness and large shear deformation of the webs, the common assumption that plane sections remain plane may no longer be valid. The diaphragms may have significant effects by constraining the web shear deformation in its vicinity. The behaviour of this type of bridges is similar to that of a sandwich beam with thick facings. This paper extends the sandwich beam theory to predict the elastic bending behaviour of such bridges. In addition, the effects of diaphragm are considered by imposing additional boundary or continuity conditions at the locations of diaphragms. A dimensionless parameter is proposed to quantify the diaphragm effects. A  $C^1$  three-node beam finite element based on the extended sandwich beam theory is developed. A numerical example is given to illustrate the theory proposed and the diaphragm effects. The results agree well with those obtained by 3-dimensional finite element method. Some tests have been carried out for verification. It is found that (a) the extended sandwich beam theory is applicable to this type of bridges; and (b) the diaphragms have significant effects on deflection, stress distribution and resultants; and (c) the effects of interaction and diaphragm may cause local failure of flanges.

**Keywords:** beam finite element; corrugated steel web; diaphragm effects; prestressed concrete; sandwich beam theory.

### 1. Introduction

The prestressed concrete bridge with corrugated steel web(s) is becoming popular worldwide. Under loading the rotation of the deformed flange sections may differ from that of the web section, and therefore the assumption that plane sections remain plane may be violated. To model the behaviour, various researchers have developed some models, e.g. Machindamrong *et al.* [1]. However, few research publications have considered the diaphragm effects. As the axial stiffness of a corrugated steel web is negligible but its shear deformation is remarkable, the diaphragms have significant effects on the behaviour of the bridge by constraining the web shear deformation in its vicinity and preventing relative longitudinal movement of the flanges as shown in Fig. 1. The behaviour of this type of bridge is similar to that of a sandwich beam with thick facings. This paper extends the sandwich beam theory [2] to predict the elastic behaviour of the bridge taking into account the diaphragm effects.

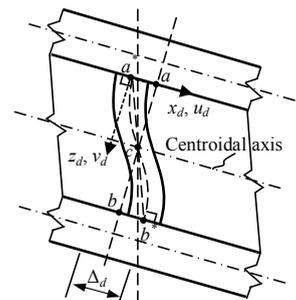


Fig. 1 Shear deformation in the vicinity of a diaphragm

### 2. An extended sandwich beam theory

For this type of bridges, the bending deflection  $v_1$  and shear deformation  $v_2$  cannot be superimposed without considering their interaction, as illustrated by the simply-supported bridge under a point

load at mid-span. Ignoring the local bending stiffness of flanges initially, shear deformation  $v_2$  would occur as shown in Fig. 2, which would induce excessive local bending stresses in the flanges in view of the kink at mid-span. It is impossible for the flanges to bend to an infinite curvature at mid-span. Instead, the flanges bend locally and smooth out the sharp discontinuity of the shear deflection. Actually interaction does occur thereby resulting in additional local curvature  $v_2''$  of the flanges, giving rise to additional shear force and bending moment. By considering the equilibrium and compatibility of an arbitrary beam segment, a general governing equation can be obtained. The presence of diaphragms can be considered by imposing additional constraints against relative longitudinal displacement of the flanges.

A  $C^1$  three-node finite element is formulated based on the sandwich beam theory proposed. Using the variational principle, an additional stiffness matrix to model the diaphragm effects can be obtained for assembly with the global stiffness matrix. If the diaphragm is thick, it may be more appropriate to consider it as a beam segment with solid section along the span with only minor modifications in modelling.

For example, a simply-supported bridge under a point load at mid-span is considered. Due to the effects of interaction and diaphragms, the deflection is reduced, and additional bending moment and shear force are induced in the flanges.

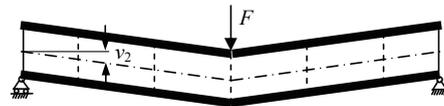


Fig. 2 Shear deformation ignoring local bending of flanges of a simply supported bridge

### 3. Experimental programme and results

A bridge specimen was fabricated for testing as shown in Fig. 3. It is observed that the shear stress of web varies along the span. The violation of the assumption of linear stress distribution across section depth is observed in the vicinity of loaded points and diaphragms.

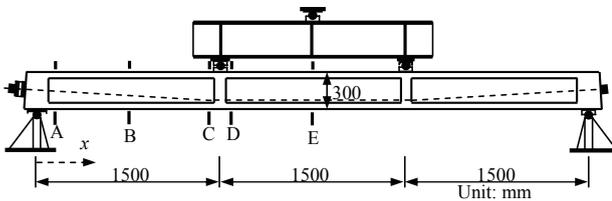


Fig. 3 Test setup of a bridge specimen

### 4. Conclusions

The extended sandwich beam theory can account for the elastic behaviour of such bridges. The common assumption that plane sections remain plane is violated. The diaphragms may have significant effects on deflection, stress distribution and resultants. The stress concentration in flanges due to the effects of interaction and diaphragms should be examined carefully, especially in the vicinity of point loads and diaphragms of short bridges.

### Acknowledgements

The work reported in this paper is supported by the Research Grants Council (RGC) of the Hong Kong Special Administrative Region, China (RGC Project No. HKU 710111E) and partially by the 2014 exchange programme between colleges and universities in Hong Kong and Mainland China.

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