The Crack Sliding Model – Unified Approach to Shear and Punching Shear

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Summary

This paper presents a unified mechanical model – The Crack Sliding Model - which can be used to calculate the shear strength as well as the punching shear strength of reinforced concrete structures. The model is based on the upper bound plasticity theorem. Contrary to the classical plasticity models to shear problems, the present model makes a clear distinction between sliding yield lines formed in un-cracked concrete and sliding yield lines formed in cracks. The basic assumptions of the model will be described and the application of the model to non-shear reinforced beams and two-way spanning slabs is outlined. Comparison of model with punching shear tests is shown. Good agreement has been found.

Keywords: Concrete, upper bound plasticity approach, crack sliding, shear, punching shear.

1. Introduction

Shear and punching shear in structural concrete are problems that have been, and still are, the subject for a vast number of research projects. This is reflected in the many models found in the literature; the reader is referred to the full length version of this paper for literature references. Even though beam shear and punching shear are variations of the same phenomenon, the two problems are usually treated separately, both in design codes as well as in research. This paper describes a unified mechanical model – The Crack Sliding Model - which can be used to calculate the shear strength as well as the punching strength of reinforced concrete structures.

2. The Crack Sliding Model

The Crack Sliding Model is an upper bound plasticity approach. Contrary to the classical plasticity approach, the Crack Sliding Model makes a clear distinction between yield lines formed in un-cracked concrete and yield lines formed in cracked concrete. This distinction is necessary because the sliding resistance in a crack is smaller than the sliding resistance of un-cracked concrete. Two Modified Coulomb failure criteria are introduced in order to distinct between cracks and un-cracked concrete. For un-cracked concrete the angle of friction and the cohesion are taken to be $\varphi = 37^{o}$ and $c = 0.25\nu_0 f_c$. Here f_c is the uniaxial compressive strength and ν_0 is an effectiveness factor taking into account softening and micro-cracking. In planes where cracks are developed, the original failure criterion is assumed to shrink such that the tensile strength disappears and the cohesion is reduced to $c' = 0.125\nu_0 f_c$. The phenomenon of crack sliding has been observed by Muttoni, who measured the relative displacements along a crack at different loading levels. When the crack is formed, the relative displacement is mainly perpendicular to the crack. At the load level corresponding to failure there is a displacement component parallel to the crack. Using a plasticity interpretation, the crack is transforming into a yield line. The observations by Muttoni are idealized in Fig. 1(b) showing formation of crack (displacement field w) followed by sliding shear failure (displacement field δ).

The calculation procedure for beams without shear reinforcement is as follows. A diagonal crack with the horizontal projection x is assumed to develop. Using a simple equivalent plastic stress distribution, the load P_{cr} required to form the crack is calculated. After formation of the crack, the longitudinal reinforcement is activated and prevents further opening of the crack. A shear failure in this crack can only take place if the sliding resistance in the crack equals P_{cr}. The sliding resistance P_u may be calculated using the plastic upper bound method. Variation of P_{cr} and P_u is schematically

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shown in Fig. 1(b). The true shear capacity is found as the intersection point between the two curves.

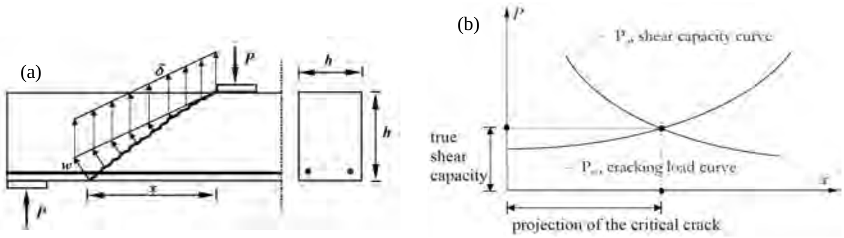

Fig. 1: (a) formation of diagonal crack followed by sliding failure, (b) shear capacity determined by intersection point of P_{cr}-curve and P_u-curve.

The concept outlined can also be applied to punching shear in two-way spanning slabs. Here P_{cr} is calculated from a cracking mechanism involving a system of inclined and vertical cracks, Fig. 2(a). The sliding resistance P_u of the inclined cracks is calculated by considering the punching mechanism illustrated in Fig. 2 (b) and (c). The true punching capacity is then determined by solving the equation $P_{cr}(x) = P_u(x)$, see also illustration in Fig. 1(b). Fig. 3 depicts comparison of calculations and test results. The agreement is satisfactory.

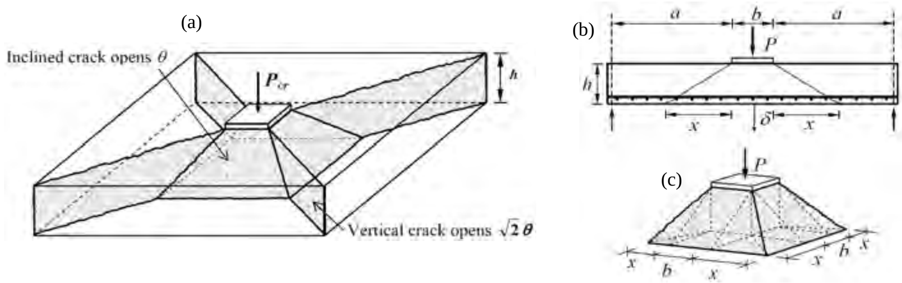

Fig. 2: Cracking mechanism and punching mechanism in a two-way spanning slab.

3. Discussions

Beam shear as well as punching shear can be treated within the framework of the model outlined. This is considered as an advantage in comparison to many other existing models which are specifically developed for either shear in beams or punching shear in slabs. For the case of beam shear, it can be shown that the model also covers continuous beams and beams with compressive normal forces or prestressing. Punching shear in circular slabs and slabs with restraining moments at the boundaries may be treated as well. This is another advantage of the model as most existing punching models either assume quadratic or circular slabs and extension to other cases are done empirically. The presented model may still be improved and further developed. Extension of the model to deal with slabs with shear reinforcement and slabs with corner- and edge loads are presently being conducted at the University of Southern Denmark

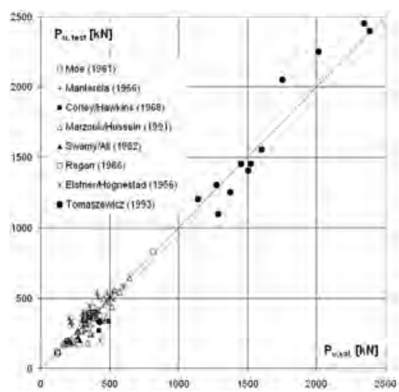

Fig. 3: Tests with quadratic slabs.